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Effects of the scale-dependent vacuum expectation values in the renormalisation group analysis of neutrino masses

N.N. Singh[a]

Department of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK

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Abstract. The contribution of scale-dependent vacuum expectation values (VEVs) of Higgs scalars, which gives significant effects in the evolution of the fundamental fermion masses in the minimal supersymmetric standard model (MSSM), is now considered in the derivation of the analytic one-loop expression for the evolution of the left-handed Majorana neutrino masses with energies. The inclusion of such an effect of the running VEV increases the stability of the neutrino masses under quantum corrections even for the low values of $\tan\beta \geq 1.42$ at the scale $\mu = 10^{12}$ GeV, and leads to a mild decrease of the neutrino masses with higher energies. Such a trend is common with that of other fundamental fermion masses.

In recent years a large number of theoretical papers were devoted to building models for generating small neutrino masses and lepton mixings within or outside the framework of the grand unified theories (GUTs) with extended $U(1)$ group [1]. Both analytic and numerical studies [2–4] have been carried out for checking the stability of the textures of the neutrino mass matrix and lepton mixing matrix under radiative quantum corrections [5]. There are basically two approaches: the top-down approach [2] which predicts the neutrino masses and mixings in terms of GUT-parameters, and the bottom-up approach [6] which predicts the running parameters at higher scales in terms of experimentally determined values at low energies. In the top-down programme, one usually starts with the running of a set of the RGEs for Yukawa matrices and gauge couplings in the MSSM (or SM), with three right-handed heavy neutrinos, taking into account the effects of the heavy neutrino mass thresholds, from the GUT scale down to the lightest right-handed neutrino mass scale (M_{R1}). This fixes the left-handed Majorana neutrino mass matrix $m_{\mathrm{LL}}(M_{\mathrm{R1}})$ through the see-saw mechanism [7] at this scale,

$$m_{\mathrm{LL}}(M_{\mathrm{R1}}) = v_u^2 Y_\nu(M_{\mathrm{R1}}) M_{\mathrm{RR}}^{-1} Y_\nu^{\mathrm{T}}(M_{\mathrm{R1}}). \quad (1)$$

Below this scale, M_{R1}, the right-handed neutrinos decouple from the theory, and the neutrino mass matrix in (1) is taken as [2]

$$m_{\mathrm{LL}}(M_{\mathrm{R1}}) = v_u^2 \kappa(M_{\mathrm{R1}}), \quad (2)$$

where κ is the coefficient of the dimension 5 neutrino mass operator. In the energy range from M_{R1} down to low energy at m_t, the running of the coefficient κ in the diagonal charged lepton basis fixes the neutrino mass matrix at scale m_t,

$$m_{\mathrm{LL}}(m_t) = v_u^2 \kappa(m_t). \quad (3)$$

In the above discussion only the scale dependence of κ is considered, and not the running of the vacuum expectation value (VEV), v_u, in (1)–(3). This led to the increase of the neutrino mass eigenvalues with energy scales, giving a significant effect for low $\tan\beta$ values. As it is strongly $\tan\beta$-dependent, this effect may lead to the instability of the neutrino masses under radiative quantum corrections[1]. For higher values of $\tan\beta$ the stability is again improved. Such an increasing trend of neutrino mass eigenvalues with the increase in energies is opposite to that of the general trend shown by other fundamental fermions (charged leptons and quarks) [8,9]. The effects of the contributions of the scale-dependent vacuum expectation values (VEVs) of Higgs scalars in the analytic one-loop expressions in the evolution of quarks and charged leptons masses at higher energies in the MSSM have been studied in [8], and this effect is quite significant.

In this paper we study the stability of the magnitudes of neutrino masses at low $\tan\beta$ and their running behaviour at different energies, by considering the scale dependence [10] of the vacuum expectation value (VEV), v_u, along with that of κ. The expression in (3) is now modified to

$$m_{\mathrm{LL}}(t) = v_u^2(t)\kappa(t), \quad (4)$$

[a] *Permanent address:* Department of Physics, Gauhati University, Guwahati-781014, India

[1] In [3,4] the stability condition is decided by the change in texture of the neutrino mass matrix only. Here we emphasise that a changing pattern of the overall magnitudes of the neutrino mass eigenvalues at different energies may also cause instability

where $v_u(t_0) = v_0 \sin\beta$, $v_0 = 174\,\mathrm{GeV}$, $t = \ln\mu, t_0 = \ln m_t$. The above equation (4) can be written as

$$\frac{\mathrm{d}\ln m_{\mathrm{LL}}(t)}{\mathrm{d}t} = \frac{\mathrm{d}\ln \kappa(t)}{\mathrm{d}t} + 2\frac{\mathrm{d}\ln v_u(t)}{\mathrm{d}t}, \tag{5}$$

where the second term on the right-hand side of the above equation is the contribution from the running of the VEV. The RGEs for v_u [8,10] and κ [2,5]in the diagonal charged lepton basis, for one-loop order in MSSM, in the energy range $t \geq t_0$, are given by

$$\frac{\mathrm{d}\ln v_u}{\mathrm{d}t} = \frac{1}{16\pi^2}\left[\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - 3h_t^2\right], \tag{6}$$

and

$$\frac{\mathrm{d}\ln \kappa}{\mathrm{d}t} = -\frac{1}{16\pi^2}\left[\frac{6}{5}g_1^2 + 6g_2^2 - 6h_t^2 - \delta_{i3}h_\tau^2 - \delta_{3j}h_\tau^2\right], \tag{7}$$

respectively. Substitution of (6) and (7) in (5) gives

$$\frac{\mathrm{d}\ln m_{\mathrm{LL}}}{\mathrm{d}t} = \frac{1}{16\pi^2}\left[-\frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + \delta_{i3}h_\tau^2 + \delta_{3j}h_\tau^2\right]. \tag{8}$$

Upon integration from low scale $t_0 = \ln m_t$ to high scale $t_{\mathrm{R1}} = \ln M_{\mathrm{R1}}$ where $t_{\mathrm{R1}} \geq t_0$, we get the correct expression for the neutrino mass matrix at t_0,

$$\frac{(m_{\mathrm{LL}}(t_0))_{ij}}{(m_{\mathrm{LL}}(t_{\mathrm{R1}}))_{ij}} = \mathrm{e}^{((9/10)I_{g1}+(9/2)I_{g2})}\mathrm{e}^{-I_\tau(\delta_{i3}+\delta_{3j})}, \tag{9}$$

$$I_f = \frac{1}{16\pi^2}\int_{\ln m_t}^{\ln M_{\mathrm{R1}}} h_f^2(t)\mathrm{d}t, \tag{10}$$

$$I_{g_i} = \frac{1}{16\pi^2}\int_{\ln m_t}^{\ln M_{\mathrm{R1}}} g_i^2(t)\mathrm{d}t \simeq \ln\left(\frac{g_i(t_{\mathrm{R1}})}{g_i(t_0)}\right)^{(1/b_i)}, \tag{11}$$

where $f = t, \tau$; $i = 1, 2, 3$, and $b_i = (33/5, 1, -3)$ for the MSSM. The correct expression in (9) will certainly affect the earlier numerical results obtained without taking into account the effect of the running VEV [2] at scale M_{R1}.

For simplicity we now follow the analysis of the RGEs for the neutrino mass eigenvalues [4]. With the inclusion of such a scale dependence of the VEV in (6), the RGEs for the mass eigenvalues given in [4] in the diagonal charged lepton basis is now modified to

$$\frac{\mathrm{d}\ln m_{\nu a}}{\mathrm{d}t} = \frac{1}{16\pi^2}\sum_{b=e,\mu,\tau}\left[-\frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 2h_b^2 V_{ba}^2\right], \tag{12}$$

where $a = 1, 2, 3$, and V_{ba} is the MNS mixing matrix element. The correct expression for the neutrino mass ratio at different energy scales is also obtained by integrating (12):

$$R_a(t_{\mathrm{R1}}) = \frac{m_{\nu a}(t_{\mathrm{R1}})}{m_{\nu a}(t_0)} \approx \mathrm{e}^{-((9/10)I_{g1}+(9/2)I_{g2})}\mathrm{e}^{2V_{\tau a}^2 I_\tau}. \tag{13}$$

In getting (13) we have neglected very small effects due to $I_{\mu,e}$ compared to I_τ, and also we assumed that $V_{\tau a}$ does not change much in the integration range[2]. For a typical value of the element of MNS mixing matrix $V_{\tau 3} \simeq 1/2^{1/2}$, we can get the condition $m_{\nu 3}(t_0) > m_{\nu 3}(t_{\mathrm{R1}})$ following (13), which shows a mild increase in neutrino masses with the decrease in energies, even for small $\tan\beta \geq 1.42$. This is due to the fact that the ratio $R_3(t_{\mathrm{R1}})$ is now independent of e^{6I_t} in the first exponential factor in (13). The same is true in (9). In fact the contribution of the running VEV effectively brings about the following replacement in the exponential factor:

$$\mathrm{e}^{-((6/5)I_{g1}+6I_{g2}-6I_t)} \to \mathrm{e}^{-((9/10)I_{g1}+(9/2)I_{g2})} \tag{14}$$

in (9) and (13).

We now study the effect of the running VEV in the evolution of the squared neutrino mass difference, $\triangle m_{ij}^2 = |m_{\nu i}^2 - m_{\nu j}^2|$ with energies. By taking the square on both sides of (13), and considering two mass eigenvalues $a = i, j$, we get approximately

$$\triangle m_{ij}^2(t_{\mathrm{R1}}) \approx \triangle m_{ij}^2(t_0)\mathrm{e}^{-2((9/10)I_{g1}+(9/2)I_{g2})}\mathrm{e}^{4V_{\tau i}^2 I_\tau}, \tag{15}$$

where we assume that the small difference between $V_{\tau i}$ and $V_{\tau j}$ for $i, j = 1, 2, 3$, does not alter much the last exponential term which can approximately be taken as $\mathrm{e}^{4V_{\tau i}^2 I_\tau} \simeq \mathrm{e}^{4V_{\tau j}^2 I_\tau} \approx 1$ for low values of I_τ. This amounts to neglecting small changes in the texture of the neutrino mass matrix which would be relevant for the evolution of the mixing angles. The evolution of $\triangle m_{ij}^2(t_{\mathrm{R1}})$ is now stable with the effects of running VEV for both low and high values of $\tan\beta$, otherwise it would have been more strongly $\tan\beta$-dependent with e^{12I_t} in the exponential factor in the case where the effect of running VEV is not included, causing more instability at low $\tan\beta$ values.

The running of the ratio of two neutrino mass eigenvalues, $R_{23} = m_{\nu 2}/m_{\nu 3}$ (and hence the running of RR_{23}) is independent of the effect of running VEV, so that the ratio of the ratios is

$$RR_{23}(t_{\mathrm{R1}}) = \frac{R_{23}(t_{\mathrm{R1}})}{R_{23}(t_0)} \approx \mathrm{e}^{-2\delta V_{\tau 32}^2 I_\tau}, \tag{16}$$

where

$$\delta V_{\tau 32}^2 = V_{\tau 3}^2 - V_{\tau 2}^2, \tag{17}$$

which can be either positive, negative or zero. For the positive value, $\delta V_{\tau 32}^2 > 0$ as in the hierarchical case [2], one gets the condition

$$R_{23}(t_0) \geq R_{23}(t_{\mathrm{R1}}), \tag{18}$$

which implies an increase in the neutrino mass ratio $m_{\nu 2}/m_{\nu 3}$ with a decrease in the energies [2]. If we start with degenerate neutrinos, $m_{\nu 2} = m_{\nu 3}$ at the scale M_{R1}, then we would get $m_{\nu 2} > m_{\nu 3}$ at the scale m_t. This shows that nearly degenerate neutrinos are not stable under quantum corrections [3].

[2] Such an approximation can be justified for the calculation of the mass eigenvalues and their ratios as the second exponential term in (13) gives almost 1 for low values of I_τ

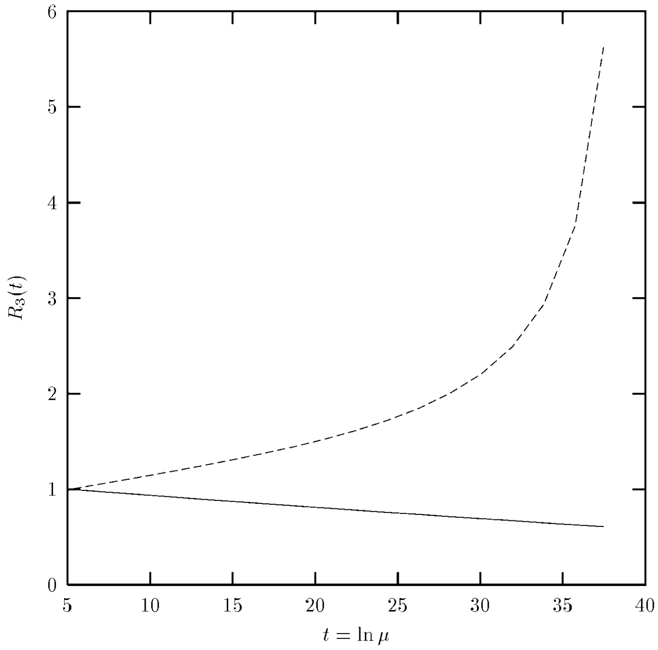

Fig. 1. Variation of $R_3(t) = m_{\nu 3}(t)/m_{\nu 3}(t_0)$ with energies $t = \ln \mu$ for small value of $\tan\beta = 1.63$. The results with and without the effect of running VEV are shown with solid line and dotted line, respectively

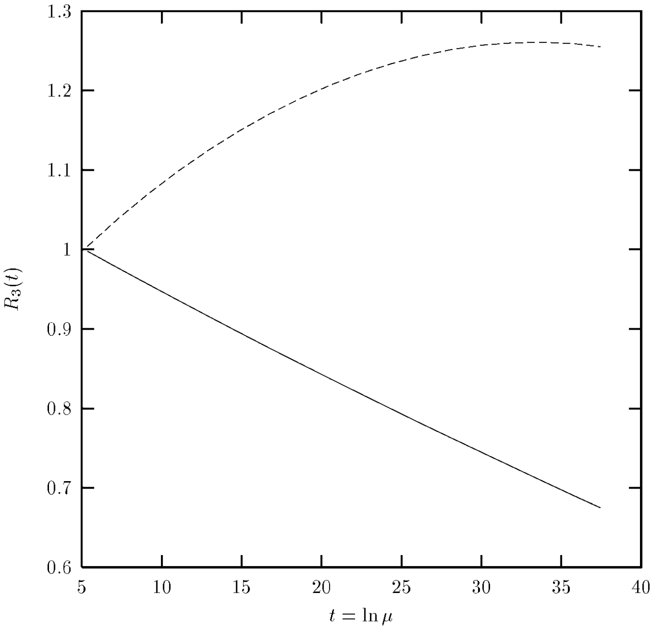

Fig. 2. Variation of $R_3(t) = m_{\nu 3}(t)/m_{\nu 3}(t_0)$ with energies $t = \ln \mu$ for large value of $\tan\beta = 57.29$. The results with and without the effect of running VEV are shown with solid line and dotted line, respectively

The above relations in (16)–(18) for $a = 2, 3$ can be generalised for any pair of mass eigenvalues $a = i, j$. For the inverted hierarchical case [2] with $m_{\nu 1} > m_{\nu 2}$, we may have $\delta V_{\tau 21}^2 < 0$, which leads to

$$R_{12}(t_0) \le R_{12}(t_{\mathrm{R1}}), \tag{19}$$

where the neutrino mass ratio $m_{\nu 1}/m_{\nu 2}$ decreases with the decrease in energies [2]. The effect of the running VEV does not change the textures of the neutrino mass matrix and hence the MNS mixing matrix.

Next we turn to a numerical analysis of the RGEs in the bottom-up approach in running from low energy scale t_0 to high energy scale, replacing t_{R1} by running t in the above equations, (9)–(19). We make use of the following input values of the running fermion masses $m_i(m_i)$ of the third family:

$$m_{t,b,\tau} = (166.5, 4.2, 1.785)\,\mathrm{GeV}, \tag{20}$$

where, for heavy flavours (top and bottom quarks) the values are derived from the input pole masses $m_t^{\mathrm{pole}} = 175.6\,\mathrm{GeV}$ [11] and $m_b^{\mathrm{pole}} = 4.7\,\mathrm{GeV}$ [12,13] using two-loop RGEs in QCD. The initial input values for the top, the bottom and τ lepton Yukawa couplings at the top quark mass scale $t_0 = \ln m_t$ in the RGEs in MSSM are usually obtained as

$$\begin{aligned} h_t(t_0) &= m_t/(174\sin\beta), \\ h_{b,\tau}(t_0) &= m_{b,\tau}/(174\eta_{b,\tau}\cos\beta). \end{aligned} \tag{21}$$

Using the CERN-LEP measurements at $M_Z = 91.18\,\mathrm{GeV}$,

$$\begin{aligned} \alpha_3(M_Z) = 0.118 \pm 0.004, \quad \alpha^{-1}(M_Z) = 127.9 \pm 0.1, \\ \sin^2\theta_\omega(M_Z) = 0.2313 \pm 0.0003, \end{aligned} \tag{22}$$

we obtain the values of gauge couplings at scale t_0 using one-loop RGEs, assuming the existence of a one-light Higgs doublet ($n = 1$) and five quark flavours below the m_t scale,

$$\alpha_{1,2,3}^{-1}(t_0) = 58.42, 29.67, 8.89. \tag{23}$$

The QCD–QED rescaling factors [6] are calculated to be

$$\eta_f = (1.54, 1.017), \quad f = b, \tau. \tag{24}$$

As a result of the numerical analysis of the RGEs for Yukawa and gauge couplings at two-loop level [6] in the energy range $t_0 < t < t_U$, the unification of three gauge couplings is observed at $M_U = 1.82 \times 10^{16}$ GeV. The values of Yukawa couplings (h_t, h_b, h_τ), gauge couplings and values of integrals I_i defined in (10) and (11) for the different values $\tan\beta = 1.42$–60.0 are estimated at different energy scales.

We present our numerical results in Figs. 1–4; the solid line refers to the analysis with the effects of running VEV in the present calculation (referred to as case A). We also present the corresponding results without the effect of running VEV in dotted line (referred to as case B) for comparison only. With a typical input value $V_{\tau 3} = 1/2^{1/2}$, the variation of the ratio $R_3(t)$ defined in (13), with energy scales t for the two representative values of $\tan\beta = 1.63$ and 57.29, are presented in Figs. 1 and 2, respectively. These figures show the evolutions of the neutrino mass eigenvalue $m_{\nu 3}$ with the increase in energy scale.

We observe that for the high value of $\tan\beta = 57.29$ in Fig. 2 the evolution of the ratio $R_3(t) = m_{\nu 3}(t)/m_{\nu 3}(t_0)$ is almost stable in both cases, A and B. However, for the low value of $\tan\beta = 1.63$ in Fig. 1, there is a significant increase in $R_3(t)$ at higher energies in case B. For example,

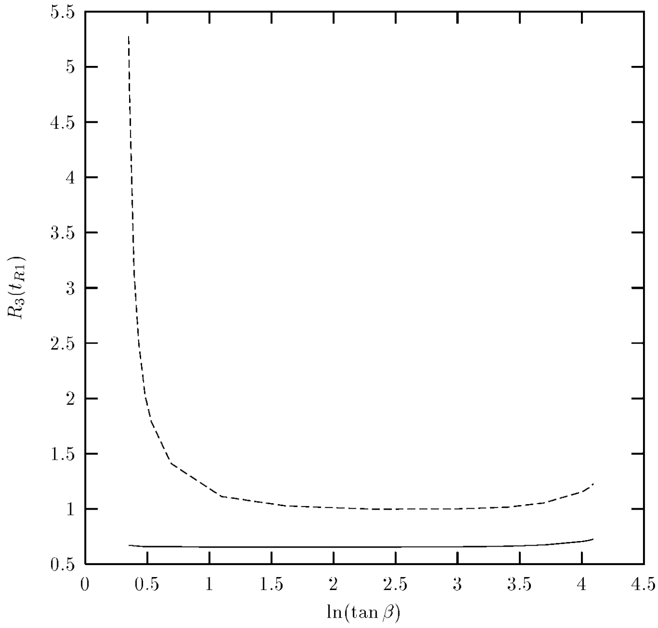

Fig. 3. Variation of $R_3(t_{\mathrm{R1}}) = m_{\nu 3}(t_{\mathrm{R1}})/m_{\nu 3}(t_0)$ with $\ln(\tan\beta)$ for $M_{\mathrm{R1}} = 10^{12}$ GeV. The results with and without the effect of running VEV are shown with solid line and dotted line, respectively

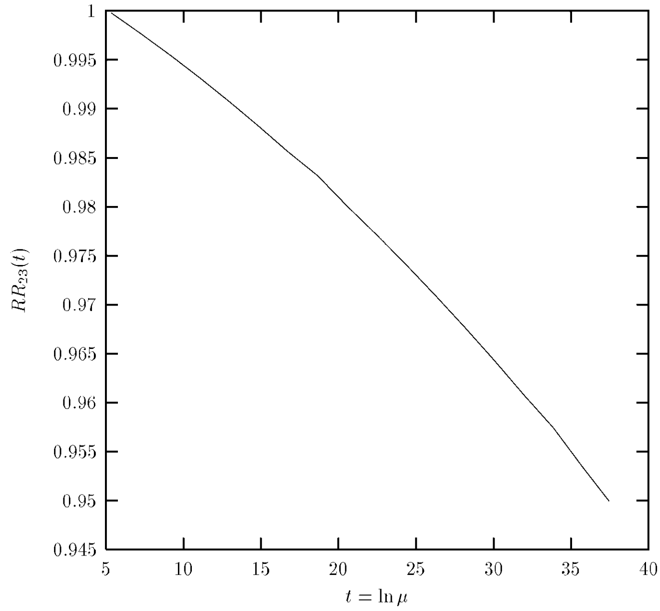

Fig. 4. Variation of the ratio of the neutrino mass ratio $RR_{23}(t) = (m_{\nu 2}(t)/m_{\nu 3}(t)) \,/\, (m_{\nu 2}(t_0)/m_{\nu 3}(t_0))$ with energies $t = \ln\mu$ for the large value of $\tan\beta = 57.29$

at $\mu = 1.82 \times 10^{16}$ GeV, the ratio $R_3(t)$ is about 5.63 in case B as shown in Fig. 1 by the dotted line. Such an unwanted feature which may cause instability is not present in case A (solid line in Fig. 1). Figure 3 shows the variation of the neutrino mass, $R_3(t_{\mathrm{R1}}) = m_{\nu 3}(t_{\mathrm{R1}})/m_{\nu 3}(t_0)$, at a particular scale, $t_{\mathrm{R1}} = 27.63$ corresponding to $M_{\mathrm{R1}} = 10^{12}$ GeV, with the different values $\tan\beta = 1.42$–60. We see that in the region of low values, $\tan\beta \geq 1.42$, there is a significant enhancement in $R_3 \leq 5.3$ in case B whereas the ratio is stable in case A for all values of $\tan\beta$. For higher values of $\tan\beta$ the ratio is again stable in case B. The same analysis is true for the cases of the other two mass eigenvalues with $a = 1, 2$. A similar analysis can be made for the evolution of $\triangle m_{ij}^2$ in (15), which would be very unstable in the low $\tan\beta$ region in case B. However, it is now stable for all values of $\tan\beta$ under radiative corrections at higher energies in case A.

Finally, we study the relative rates of the evolution of two neutrino mass eigenvalues in terms of their ratio, $R_{23} = m_{\nu 2}/m_{\nu 3}$ given in (16), in going from low to high energies. We consider the high value of $\tan\beta = 57.29$ where the effect of I_τ is large, and this ratio increases with the decrease in energies by a few percent only. This is shown in Fig. 4 where we present the evolution of the ratio of the ratios $RR_{23}(t)$ in (16) with energies. This leads to a mild increase in the hierarchical relation, $m_{\nu 2}/m_{\nu 3}$, at lower energies. As noted earlier, such hierarchical ratios are independent of the effect of the running VEV. Finally we point out the changes arising from the running of the VEV in the earlier calculations [2] of the neutrino masses. The earlier results at low scale m_t in [2] do not change at all. However if we prefer to express the neutrino mass matrix at higher scale M_{R1}, then we have to take the effect of the running VEV, $v_u(t_{\mathrm{R1}})$ in place of $v_u(t_0)$, in account, which modifies the earlier numerical results at the scale M_{R1}.

To conclude, we have considered the contributions of scale-dependent vacuum expectation values (VEVs) of Higgs scalars in deriving the analytic one-loop expression for the running of the left-handed Majorana neutrino masses with energies in the MSSM. This gives significant changes in the expression of the evolution of the neutrino masses, and also increases the stability of the neutrino masses under quantum corrections even for low $\tan\beta$. We observed a mild decreasing trend of the neutrino masses with higher energies, which is now common with that of all other fermion masses in nature.

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